



Constrained Evolutionary Optimization of the Signal Growth Rate in an RKO with an Axially Varying Waveguide Diameter

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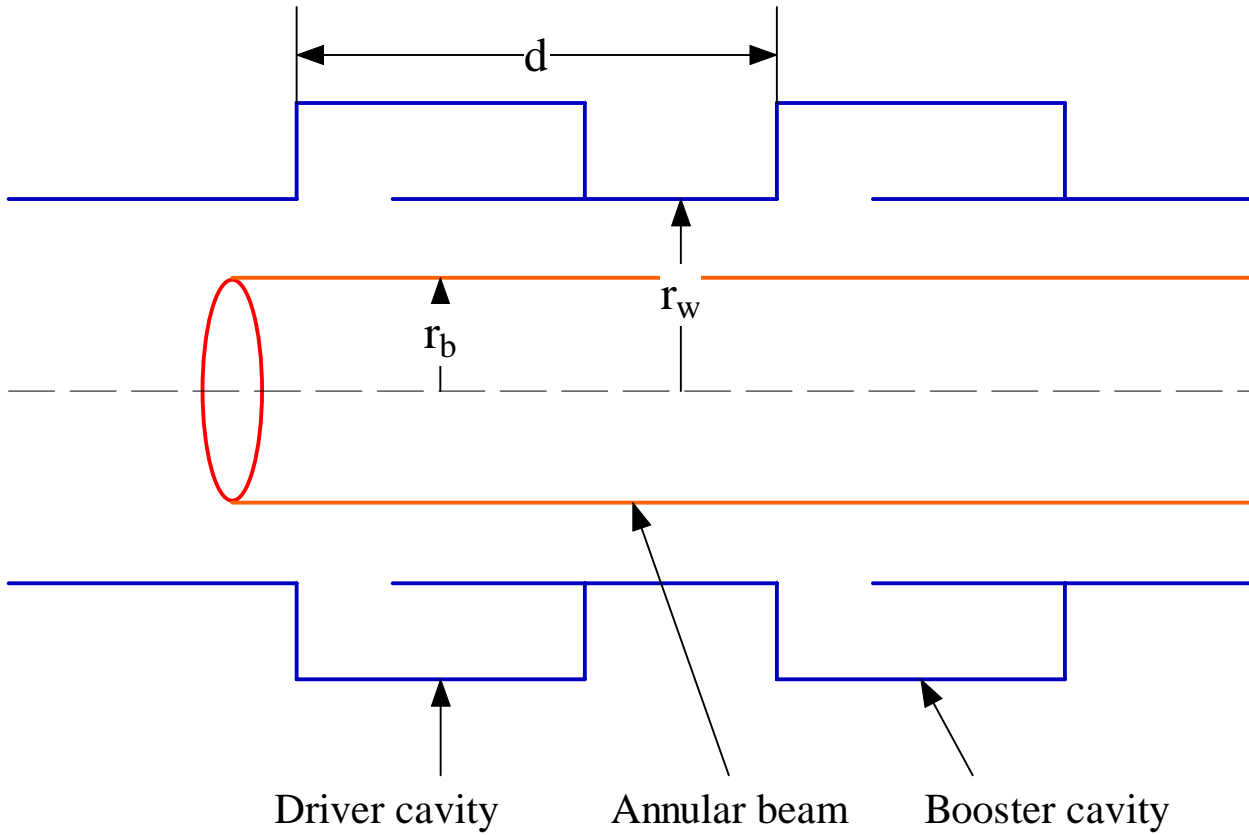


Overview

- **Background**
 - Relativistic Klystron Oscillator
 - Evolutionary Algorithms
- **Methodology**
 - Multi-cavity RKO Model
 - Computational Approach
- **Results**
- **Conclusions and Future Directions**
- **References**



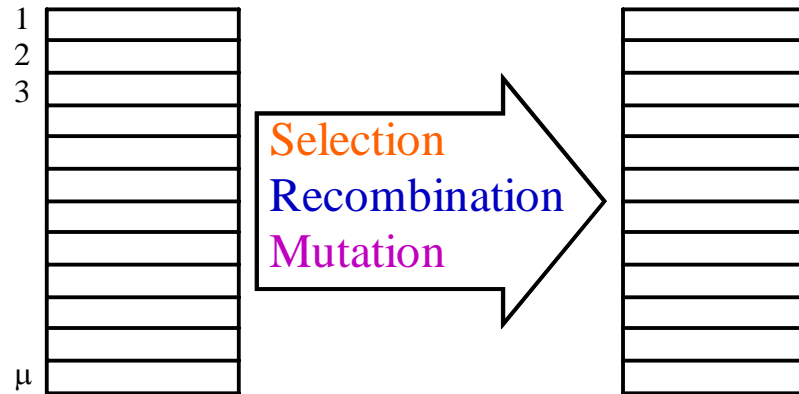
Background: RKO (Hendricks, et al., 1996)



- **Transverse electron motion restricted** by static magnetic field
- **First cavity driven** by external RF source
- RF gap voltage **modulates electron beam velocity**
- **Coupled booster cavity** enhances AC component (Luginsland, et al, 1996)



Background: Evolutionary Algorithms



- Inspired by processes of natural selection
- Population initialized as collection of random individuals
- Individuals evaluated according to fitness function
- Genetic operators applied to population
 - **Selection**: Offspring population biased toward more fit individuals
 - **Recombination**: Features from multiple parents combined in offspring
 - **Mutation**: Random variation added to offspring
- Applied successfully as **optimum-seeking techniques**
 - Useful for objective functions that are discontinuous, nonconvex, ...



Methodology: Multi-cavity RKO Model

- **Model evolution of gap voltages including effects of:**
 - Cavity resonances
 - Electromagnetic coupling
 - Beam coupling
- **Assumptions**
 - Small signal, modal, steady-state solutions
 - ⇒ Superposition principle applies to beam modulation
 - Cavity coupling is weak and occurs through cutoff waveguide
 - ⇒ Only nearest neighbor electromagnetic coupling is significant
- **Generalizes Luginsland's dispersion relation model of the two-cavity RKO (Luginsland, 1996) to the N-cavity RKO**
 - Cavities may have distinct natural frequencies, qualities, and impedances
 - Drift regions may have distinct radii, lengths, and loss coefficients



Methodology: Multi-cavity RKO Model

Assuming solutions $e^{-j\omega t}$, the gap voltage V_m satisfies

$$L_m(\omega)V_m + C_{m-1}V_{m-1} + C_mV_{m+1} + \sum_{n < m} \Gamma_{m,n} V_n = 0 ,$$

where the damped harmonic oscillator operator is

$$L_m(\omega) = \frac{\omega^2}{\omega_{0,m}^2} - \frac{j\omega}{\omega_{0,m}Q_m} - 1 ,$$

the electromagnetic coupling coefficient is

$$C_m = \chi_{c,m} \exp \left[-\frac{2.405}{r_{w,m}} \sqrt{1 - \left(\frac{2\pi\omega_{0,m}r_{w,m}}{0.383c} \right)^2} (x_{m+1} - x_m) \right] ,$$

and the beam coupling coefficient is

$$\Gamma_{m,n} = \frac{Z_m}{R} \sin \left\{ \sum_{r=n}^{m-1} k_{p,r} (x_{r+1} - x_r) \right\} \exp \left[-\frac{j\omega_{0,n}}{\beta c} (x_m - x_n) \right]$$



Methodology: Multi-cavity RKO Model

- The evolution of the cavity voltages $V=(V_1, V_2, \dots, V_N)^T$ are thus described by $[A(\omega)]V = 0$, where

$$A(\omega) = \begin{bmatrix} L_1(\omega) & -C_1 & 0 & \dots & \dots & \dots & 0 \\ -\Gamma_{2,1} - C_1 & L_2(\omega) & -C_2 & 0 & \dots & \dots & 0 \\ -\Gamma_{3,1} & -\Gamma_{3,2} - C_2 & L_3(\omega) & -C_3 & 0 & \dots & 0 \\ -\Gamma_{4,1} & -\Gamma_{4,2} & -\Gamma_{4,3} - C_3 & L_4(\omega) & -C_4 & \dots & 0 \\ -\Gamma_{5,1} & -\Gamma_{5,2} & -\Gamma_{5,3} & -\Gamma_{5,4} - C_4 & L_5(\omega) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & -C_{N-1} \\ -\Gamma_{N,1} & -\Gamma_{N,2} & -\Gamma_{N,3} & -\Gamma_{N,4} & \dots & -\Gamma_{N,N-1} - C_{N-1} & L_N(\omega) \end{bmatrix}$$

- Resonant frequencies ω satisfy $\det[A(\omega)] = 0$
 - $\det[A(\omega)]$ is a polynomial of degree $2N$ in ω
 - $-\text{Im}[\omega]$ is the mode's growth rate, to be maximized



Methodology: GENOCOP III (Michalewicz, 1992)



- Public domain UNIX-based **real-valued EA** used widely and successfully for parameter optimization problems
- Minimization and maximization problems
- **Constraints:**
 - linear equality,
 - linear inequality, and
 - non-linear inequality
- **Operators:**
 - **selection:** exponential ranking
 - **crossover:** whole and simple arithmetic
 - **mutation:** uniform, boundary, non-uniform, and whole non-uniform
- Maintains **separate “reference” population** of feasible individuals; highly fit but infeasible individuals are occasionally recombined with reference individuals



Methodology: Independent Variables and Domains

Identify candidate designs

- Represented as vectors of independent variables:

$$(V_0, I_0, r_i, r_o - r_i, f_{0,1}, \dots, f_{0,N}, Q_1, \dots, Q_N, Q_1 Z_1, \dots, Q_N Z_N, d_1, \dots, d_{N-1}, \chi_{r,1}, \dots, \chi_{r,N-1}, \chi_{c,1}, \dots, \chi_{c,N-1})^T$$

- Components satisfy variable domain constraints:

Quantity	Lower bound	Variable	Upper bound
Beam voltage	300 kV	V_0	650 kV
Beam current	5 kA	I_0	35 kA
Beam inner radius	0.1 cm	r_i	12 cm
Beam thickness	0.1 cm	$r_o - r_i$	1 cm
Cavity natural frequencies	1 GHz	f_0	2 GHz
Cavity qualities	50	Q	500
Cavity impedances	50 Ohms	QZ	377 Ohms
Drift space lengths	2 cm	d	50 cm
Drift space radius multipliers	0	χ_r	1
Drift space EM coupling multipliers	0	χ_c	1



Methodology: Computational Approach

Check that drift space radius bounds satisfy constraints:

$$\left(0.95 \frac{0.383c}{f_{0,m}}\right) - (r_o + 0.2cm) \geq 0$$

Compute drift space radii:

$$r_{w,m} = \chi_{r,m} \left(0.95 \frac{0.383c}{f_{0,m}}\right) + (1 - \chi_{r,m})(r_o + 0.2cm)$$

Check that limiting currents are not exceeded:

$$17000 \left[\left(1 + \frac{V_0}{mc^2}\right)^{\frac{2}{3}} - 1 \right]^{\frac{3}{2}} \left[1 - 2 \left(\frac{r_i^2}{r_o^2 - r_i^2} \log \frac{r_o}{r_i} - \log \frac{r_w}{r_o} \right) \right]^{-1} - I_0 \geq 0$$



Methodology: Computational Approach

- **Compute electromagnetic coupling coefficients**
- **Compute beam coupling coefficients**
- **Compute harmonic operator coefficients**
- **Construct the NxN matrix $A(\omega)$**
 - Elements are polynomials in ω , represented by their coefficients
- **Reduce $A(\omega)$ to lower triangular form:**
 - For rows $i = N-1$ down to 1, and each element $[A(\omega)]_{i,j}$ in row i
 - Multiply by $[A(\omega)]_{i+1,i+1}$
 - Subtract $[A(\omega)]_{i,i+1} [A(\omega)]_{i+1,j}$
 - $\det([A(\omega)])$ is now stored in $[A(\omega)]_{1,1}$ as a polynomial in ω of degree $2N$
- **Use Laguerre's method to find roots of $\det(A[\omega])$**
- **Choose root ω s.t. $\text{Re}[\omega] > 0$ and $\text{Im}[\omega]$ is minimized**
- **Assign $\text{Im}[\omega]$ as the fitness of the candidate design**



Methodology: EA Parameters



- **Standard GENOCOP operator parameters**
 - 5 (necessarily feasible) individuals in reference population
 - 20 (possibly feasible) individuals in search population
 - 10,000 (x2) evaluations per experiment
 - ...
- **50 independent experiments => 500,000 evaluations**
- **Wall clock time (Pentium II, 233 MHz, NT) \approx 14 hours**



Results: High Growth-Rate, Non-Intuitive Designs



- **Each experiment found high growth-rate designs**
 - In comparison to a 10 cavity version of one good 2 cavity design, for which the growth rate is 1.30 nsec^{-1}
 - Best growth rate in these experiments is 2.07 nsec^{-1}
 - Enhanced growth rates of 10-cavity design allow pure oscillator operation (two-cavity design requires injection-locked operation)
- **Designs are non-intuitive (typical of EA-based design)**
 - Parameters differ significantly between cavities, and between drift spaces
- **Best designs from various experiments are dissimilar**
 - Suggests the EA designs may be far from the global optimum



Conclusions

- **Theoretical model of signal growth rate in a multi-cavity RKO developed, incorporating electromagnetic and beam coupling effects**
- **Computational model manipulates arrays of polynomials to find determinant of interaction matrix, then uses Laguerre's method to find resonant frequencies and accompanying growth rates**
- **GENOCOP, a real-valued EA, using independent linear constraints on design parameters and standard algorithm parameters, identifies designs with growth rates that are significantly higher than intuitive designs**



Future Directions



- **Perform PIC simulations of best designs**
- **Improve theoretical and computational models**
 - **Consider limiting currents at cavity gaps**
 - **Assign non-zero fitness to designs violating constraints**
 - **Reduce beam current to smallest limiting current**
 - **Reduce beam radius to fit within narrowest drift space**
 - **Consider mode competition and sensitivity to design parameters**
- **Improve effectiveness and efficiency of optimization**
 - **Hybridize with local search (e.g. conjugate gradient)**
 - **Consider other optimum-seeking techniques**
 - **Reduce the number of roots found**



References

- **Hendricks, Coleman, Lemke, Arman, and Bowers, *Physical Review Letters*, vol 76, no 154, 1996.**
- **Luginsland, Lau, Hendricks, and Coleman, *IEEE Transactions on Plasma Science*, vol 24, no 3, 1996.**
- **Michalewicz, Genetic Algorithms + Data Structures = Evolution Programs, Springer-Verlag, New York, 1992.**