Optimization of the Signal Growth Rate in a Class of Multicavity RKO with Axially Varying Geometry

Laurence D. Merkle

Department of Computer Science

USAF Academy

John W. Luginsland

Air Force Research Laboratory: Phillips Research Site

Directed Energy Directorate

Kirtland AFB, NM

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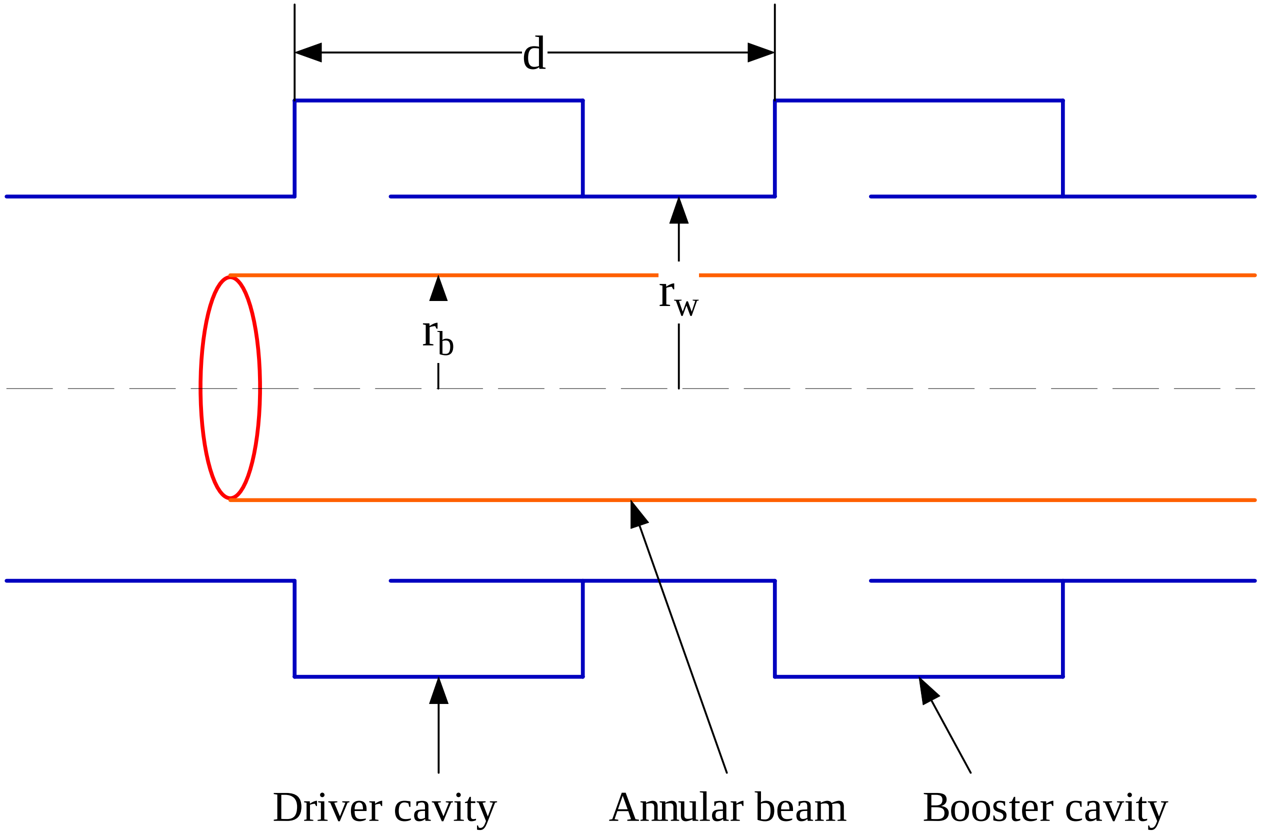
Overview

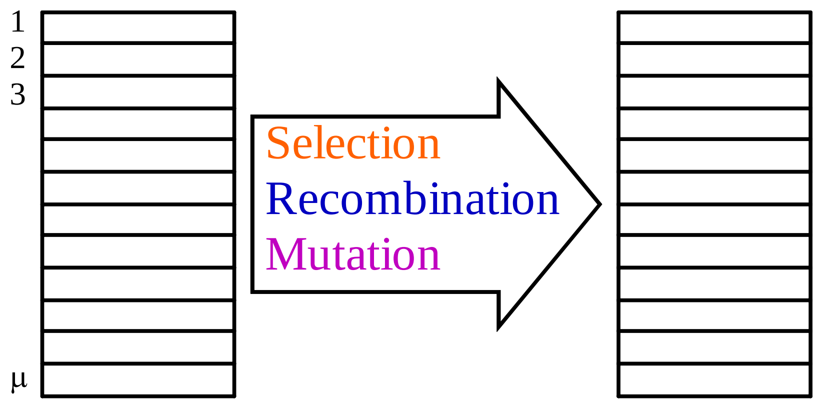
- Background
  - Relativistic Klystron Oscillator
  - Evolutionary Algorithms
- Methodology
  - Multi-cavity RKO Model
  - Computational Approach
- Results
- Conclusions and Future Directions
- References

Background: RKO
(Hendricks, et al., 1996)



- Transverse electron motion restricted by static magnetic field
- First cavity driven by external RF source
- RF gap voltage modulates electron beam velocity
- Coupled booster cavity enhances AC component (Luginsland, et al, 1996)

Background: Evolutionary Algorithms



- **Inspired by processes of natural selection**
- **Population initialized as collection of random individuals**
- **Individuals evaluated according to fitness function**
- **Genetic operators applied to population**
  - **Selection: Offspring population biased toward more fit individuals**
  - **Recombination: Features from multiple parents combined in offspring**
  - **Mutation: Random variation added to offspring**
- **Applied successfully as optimum-seeking techniques**
  - **Useful for objective functions that are discontinuous, nonconvex, ...**

Methodology:
Multi-cavity RKO Model

- **Model evolution of gap voltages including effects of:**
  - **Cavity resonances**
  - **Electromagnetic coupling**
  - **Beam coupling**
- **Assumptions**
  - **Small signal, modal, steady-state solutions**
    - $\Rightarrow$**Superposition principle applies to beam modulation**
  - **Cavity coupling is weak and occurs through cutoff waveguide**
    - $\Rightarrow$**Only nearest neighbor electromagnetic coupling is significant**
- **Generalizes Luginsland's dispersion relation model of the two-cavity RKO (Luginsland, 1996) to the N-cavity RKO**
  - **Cavities may have distinct natural frequencies, qualities, and impedances**
  - **Drift regions may have distinct radii, lengths, and loss coefficients**

Methodology: Multi-cavity RKO Model

Assuming solutions $e^{-j\omega t}$, the gap voltage V_m satisfies

$$L_m(\omega)V_m + C_{m-1}V_{m-1} + C_m V_{m+1} + \sum_{n<m} \Gamma_{m,n} V_n = 0 \ ,$$

where the damped harmonic oscillator operator is

$$L_m(\omega) = \frac{\omega^2}{\omega_{0,m}^2} - \frac{j\omega}{\omega_{0,m}Q_m} - 1 \ ,$$

the electromagnetic coupling coefficient is

$$C_m = \chi_{c,m} \exp\left[-\frac{2.405}{r_{w,m}} \sqrt{1 - \left(\frac{2\pi\omega_{0,m} r_{w,m}}{0.383c} \right)^2} \left(x_{m+1} - x_m \right) \right] \ ,$$

and the beam coupling coefficient is

$$\Gamma_{m,n} = \frac{Z_m}{R} \sin\left\{ \sum_{r=n}^{m-1} k_{p,r} \left(x_{r+1} - x_r \right) \right\} \exp\left[-\frac{j\omega_{0,n}}{\beta c} \left(x_m - x_n \right) \right]$$

Methodology: Multi-cavity RKO Model

- The evolution of the cavity voltages $V=(V_1, V_2, \ldots, V_N)^T$ are thus described by $[A(\omega)]V = 0$, where

$$A(\omega)=\begin{bmatrix} L_1(\omega) & -C_1 & 0 & \cdots & \cdots & \cdots & 0 \\ -\Gamma_{2,1}-C_1 & L_2(\omega) & -C_2 & 0 & \cdots & \cdots & 0 \\ -\Gamma_{3,1} & -\Gamma_{3,2}-C_2 & L_3(\omega) & -C_3 & 0 & \cdots & 0 \\ -\Gamma_{4,1} & -\Gamma_{4,2} & -\Gamma_{4,3}-C_3 & L_4(\omega) & -C_4 & & 0 \\ -\Gamma_{5,1} & -\Gamma_{5,2} & -\Gamma_{5,3} & -\Gamma_{5,4}-C_4 & L_5(\omega) & & 0 \\ \vdots & \vdots & \vdots & & & \ddots & -C_{N-1} \\ -\Gamma_{N,1} & -\Gamma_{N,2} & -\Gamma_{N,3} & -\Gamma_{N,4} & \cdots & -\Gamma_{N,N-1}-C_{N-1} & L_N(\omega) \end{bmatrix}$$

- Resonant frequencies ω satisfy $\det[A(\omega)] = 0$
  - $\det[A(\omega)]$ is a polynomial of degree 2N in ω
  - $-\mathrm{Im}[\omega]$ is the mode's growth rate, to be maximized

Methodology: GENOCOP III
(Michalewicz, 1992)

- Public domain UNIX-based real-valued EA used widely and successfully for parameter optimization problems
- Minimization and maximization problems
- Constraints:
  - linear equality,
  - linear inequality, and
  - non-linear inequality
- Operators:
  - selection: exponential ranking
  - crossover: whole and simple arithmetic
  - mutation: uniform, boundary, non-uniform, and whole non-uniform
- Maintains separate “reference” population of feasible individuals; highly fit but infeasible individuals are occasionally recombined with reference individuals

Methodology: Independent Variables and Domains

Identify candidate designs

- **Represented as vectors of independent variables:**

$$(V_0, I_0, r_i, r_o\text{-}r_i, f_{0,1},\ldots,f_{0,N}, Q_1,\ldots,Q_N, Q_1Z_1,\ldots,Q_NZ_N, d_1,\ldots,d_{N-1}, \chi_{r,1},\ldots,\chi_{r,N-1}, \chi_{c,1},\ldots,\chi_{c,N-1})^T$$

- **Components satisfy variable domain constraints:**

Quantity	Lower bound	Variable	Upper bound
Beam voltage	300 kV	V_0	650 kV
Beam current	5 kA	I_0	35 kA
Beam inner radius	0.1 cm	r_i	12 cm
Beam thickness	0.1 cm	$r_o - r_i$	1 cm
Cavity natural frequencies	1 GHz	f_0	2 GHz
Cavity qualities	50	Q	500
Cavity impedances	50 Ohms	QZ	377 Ohms
Drift space lengths	2 cm	d	50 cm
Drift space radius multipliers	0	χ_r	1
Drift space EM coupling multipliers	0	χ_c	1

Methodology: Computational Approach

Check that drift space radius bounds satisfy constraints:

$$\left(0.95\frac{0.383c}{f_{0,m}}\right)-(r_o+0.2cm) \;\; \geq \;\; 0$$

Compute drift space radii:

$$r_{w,m}=\chi_{r,m}\left(0.95\frac{0.383c}{f_{0,m}}\right)+(1-\chi_{r,m})(r_o+0.2cm)$$

Check that limiting currents are not exceeded:

$$17000\left[\left(1+\frac{V_0}{mc^2}\right)^{\frac{2}{3}}-1\right]^{\frac{3}{2}}\left[1-2\left(\frac{r_i^2}{r_o^2-r_i^2}\log\frac{r_o}{r_i}-\log\frac{r_w}{r_0}\right)\right]^{-1}-I_0 \;\geq\; 0$$

Methodology: Computational Approach

- **Compute electromagnetic coupling coefficients**
- **Compute beam coupling coefficients**
- **Compute harmonic operator coefficients**
- **Construct the NxN matrix $A(\omega)$**
  - **Elements are polynomials in ω, represented by their coefficients**
- **Reduce $A(\omega)$ to lower triangular form:**
  - **For rows i = N-1 down to 1, and each element $[A(\omega)]_{i,j}$ in row i**
    - **Multiply by $[A(\omega)]_{i+1,i+1}$**
    - **Subtract $[A(\omega)]_{i,i+1}$ $[A(\omega)]_{i+1,j}$**
  - **det ($[A(\omega)]$) is now stored in $[A(\omega)]_{1,1}$ as a polynomial in ω of degree 2N**
- **Use Laguerre's method to find roots of det($A[\omega)]$)**
- **Choose root ω s.t. Re[ω] > 0 and Im[ω] is minimized**
- **Assign Im[ω] as the fitness of the candidate design**

Methodology: EA Parameters

- Standard GENOCOP operator parameters
  - 5 (necessarily feasible) individuals in reference population
  - 20 (possibly feasible) individuals in search population
  - 10,000 (x2) evaluations per experiment
  - ...
- 50 independent experiments => 500,000 evaluations
- Wall clock time (Pentium II, 233 MHz, NT) ≈ 14 hours

Results: High Growth-Rate, Non-Intuitive Designs

- **Each experiment found high growth-rate designs**
  - **In comparison to a 10 cavity version of one good 2 cavity design, for which the growth rate is 1.30 $nsec^{-1}$**
  - **Best growth rate in these experiments is 2.07 $nsec^{-1}$**
  - **Enhanced growth rates of 10-cavity design allow pure oscillator operation (two-cavity design requires injection-locked operation)**
- **Designs are non-intuitive (typical of EA-based design)**
  - **Parameters differ significantly between cavities, and between drift spaces**
- **Best designs from various experiments are dissimilar**
  - **Suggests the EA designs may be far from the global optimum**

Conclusions

- **Theoretical model of signal growth rate in a multi-cavity RKO developed, incorporating electromagnetic and beam coupling effects**
- **Computational model manipulates arrays of polynomials to find determinant of interaction matrix, then uses Laguerre’s method to find resonant frequencies and accompanying growth rates**
- **GENOCOP, a real-valued EA, using independent linear constraints on design parameters and standard algorithm parameters, identifies designs with growth rates that are significantly higher than intuitive designs**

Future Directions

- Perform PIC simulations of best designs
- Improve theoretical and computational models
  - Consider limiting currents at cavity gaps
  - Assign non-zero fitness to designs violating constraints
    - Reduce beam current to smallest limiting current
    - Reduce beam radius to fit within narrowest drift space
  - Consider mode competition and sensitivity to design parameters
- Improve effectiveness and efficiency of optimization
  - Hybridize with local search (e.g. conjugate gradient)
  - Consider other optimum-seeking techniques
  - Reduce the number of roots found

References

- **Hendricks, Coleman, Lemke, Arman, and Bowers, *Physical Review Letters*, vol 76, no 154, 1996.**
- **Luginsland, Lau, Hendricks, and Coleman, *IEEE Transactions on Plasma Science*, vol 24, no 3, 1996.**
- **Michalewicz, <u>Genetic Algorithms + Data Structures = Evolution Programs</u>, Springer-Verlag, New York, 1992.**