Design Optimization for a Novel Class of High Power Microwave Sources: Incorporating Constraints in a Real-Valued Evolutionary Algorithm

Laurence D. Merkle

Department of Computer Science, United States Air Force Academy

- The relativistic klystron oscillator (RKO) is a high power microwave (HPM) source in which the kinetic energy of a relativistic electron beam is converted into coherent microwave radiation. Two theoretical models are developed relating the growth rate of the microwave output power to the design parameters. One of the models generalizes easily to a novel multi-cavity class of RKO devices, which has significantly better growth rates than standard two-cavity RKOs.
- Optimization of the growth rate via analytical and standard numerical techniques is intractable because of the existence of many local optima. Instead, the growth rate is optimized using a real-valued evolutionary algorithm (EA), which performs mutation, selection, and recombination on a population of candidate design parameters.
- The design space of the McRKO is subject to a number of nonlinear constraints. Several methods of incorporating these non-linear constraints in the EA are compared, including a penalty function, a “blind” repair mechanism, and a domain specific repair mechanism. The latter is shown to result in significantly better designs.

Presented at Rose-Hulman Institute of Technology, March 15, 2002

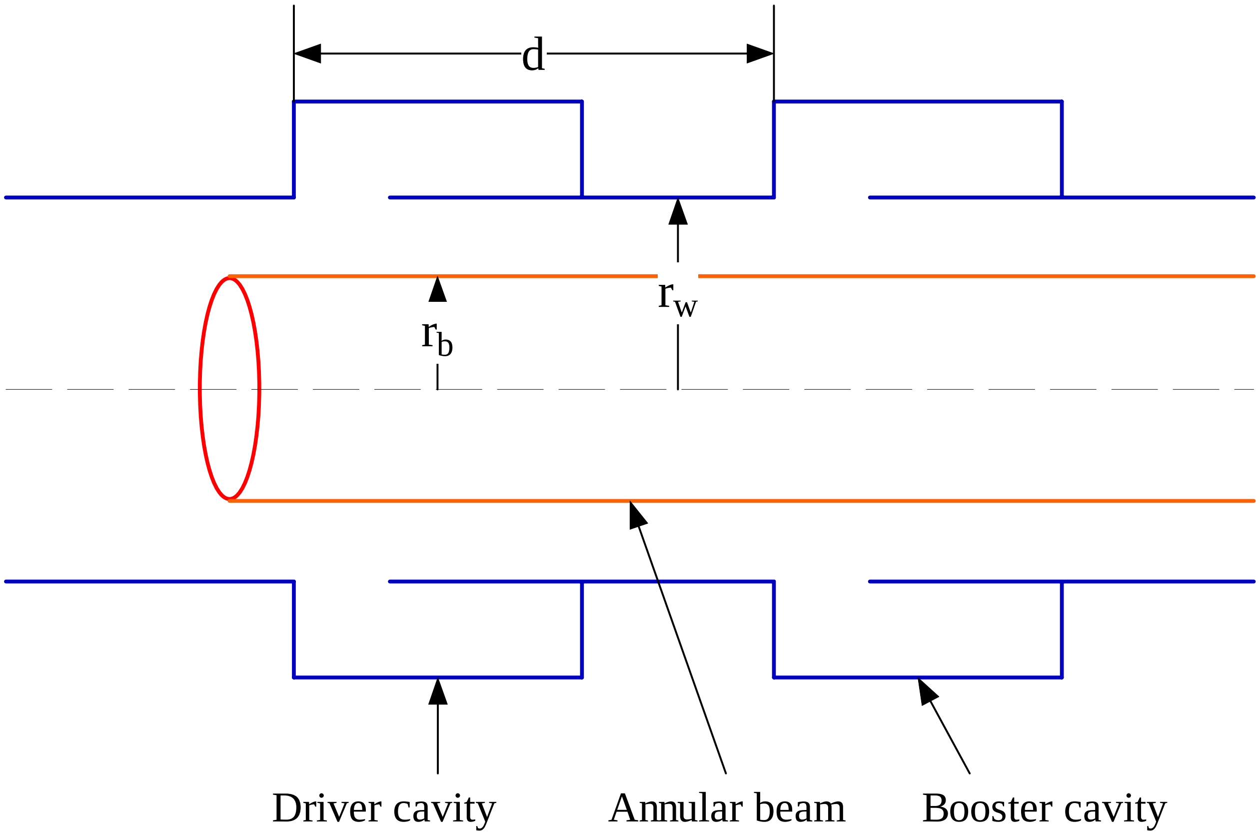
Acknowledgements

- **In collaboration with**
  Dr. John Luginsland, Air Force Research Laboratory
  (now with SAIC)
- **Sponsored by**
  Air Force Research Laboratory: Phillips Research Site
  Directed Energy Directorate
  Kirtland AFB, NM
- **Portions presented at**
  - American Physical Society Division of Plasma Physics 1998 Meeting, November 18, 1998, New Orleans, LA
  - 1999 IEEE International Conference on Plasma Science June 22, 1999, Monterey, CA
  - 2000 IEEE International Conference on Plasma Science June 7, 2000, New Orleans, LA

Overview

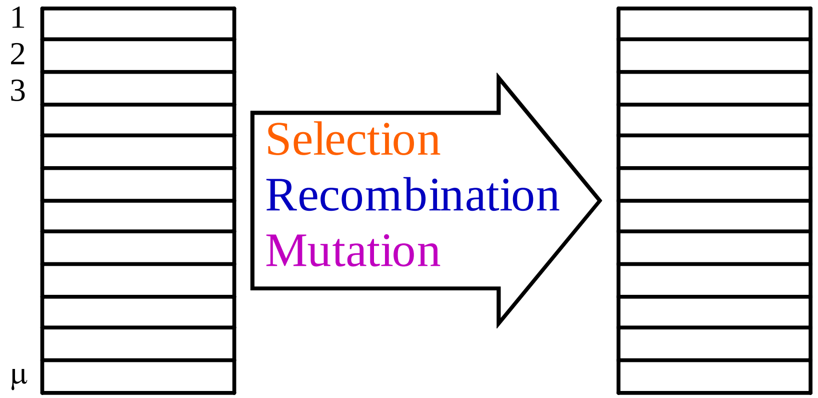
- Background
  - Relativistic Klystron Oscillator
  - Evolutionary Algorithms
- Methodology
  - Signal Growth Rate Models
  - Computational Approach
    - Handling Non-linear Constraints
- Results
- Conclusions and Future Directions
- References

Background: RKO (Hendricks, et al., 1996)



- High-power microwave source
- Transverse electron motion restricted by static magnetic field
- First cavity driven by external RF source
- RF gap voltage modulates electron beam velocity
- Coupled booster cavity enhances AC component (Luginsland, et al, 1996)

Background: Evolutionary Algorithms

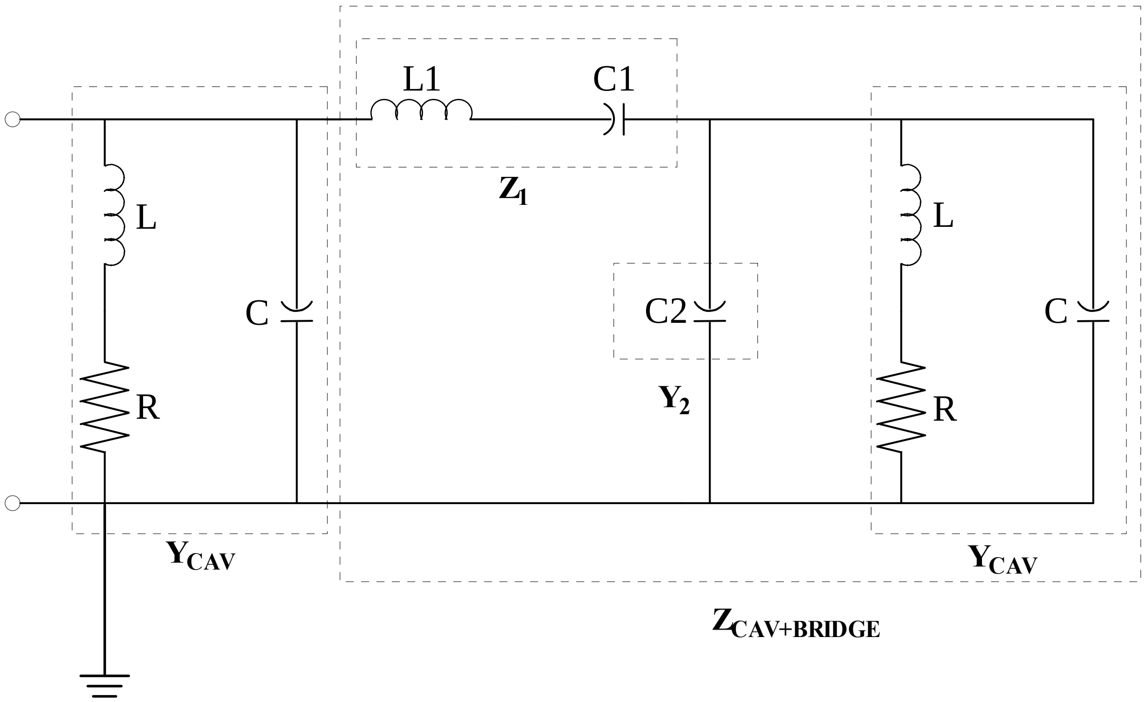


- **Inspired by processes of natural selection.**
- **Population initialized as collection of random individuals.**
- **Individuals evaluated according to fitness function.**
- **Genetic operators applied to population.**
  - **Selection: Offspring population biased toward more fit individuals.**
  - **Recombination: Features from multiple parents combined in offspring.**
  - **Mutation: Random variation added to offspring.**
- **Applied successfully as optimum-seeking techniques.**
  - **Useful for objective functions that are discontinuous, nonconvex, ...**

Background: GENOCOP (Michalewicz, 1992)

- Public domain, UNIX-based, real-valued EA
- Used widely and successfully for parameter optimization problems
  - Offers a wide variety of selection, recombination, and mutation operators shown to be effective in practice
- Supports constrained minimization and maximization problems
  - Constraint types: non-linear equality, linear and non-linear inequality, domain constraints
  - Maintains separate reference population of individuals satisfying the specified constraints
  - Highly fit search individuals are occasionally recombined with reference individuals to produce new reference individuals

Methodology: RKO Circuit Model (Schiffler, et al., 1998)



- Bridge parameters L_1, C_1, and C_2 determined by gap separation based on energy principles
- "Cold tube" resonant frequencies ω found from Im[Y] = 0

$$Z_1 = j\omega L_1 + 1/(j\omega C_1)$$
$$Y_2 = j\omega C_2$$
$$Y_{CAV} = j\omega C + 1/(R + j\omega L)$$

$$Z_{CAV+BRIDGE} = Z_1 + 1/(Y_{CAV} + Y_2)$$
$$Y = Y_{CAV} + 1/Z_{CAV+BRIDGE}$$

Methodology: RKO Growth Rate (Luginsland, et al., 1996)

Coupling constant C determined by the ratio Re[ω/ω_0] of the cold tube resonant frequency to the natural frequency and the cold tube growth rate relationship

$$\omega = \omega_0 \left[1 + \frac{j}{2Q} \pm \frac{C}{2} \right]$$

Beam voltage V_0 and current I_0 determine parameters R and Z; together with gap separation d determine θ and k_p. Growth rate Im[ω] is determined by

$$\omega = \omega_0 \left[1 + \frac{j}{2Q} \pm \frac{C}{2} \sqrt{1 + \frac{Z}{CR} \sin(k_p d) e^{-j\theta}} \right]$$

Methodology: Independent Variables and Domains

- **Limited set of parameters varied:**
  - **Beam voltage V_0**
  - **Beam current I_0**
  - **Gap separation d**
- **Independent linear constraints (domains):**
  - **$400 \text{ kV} \leq V_0 \leq 650 \text{ kV}$**
  - **$5 \text{ kA} \leq I_0 \leq 35 \text{ kA}$**
  - **$2 \text{ cm} \leq d \leq 50 \text{ cm}$**

Methodology: GENOCOP Operators

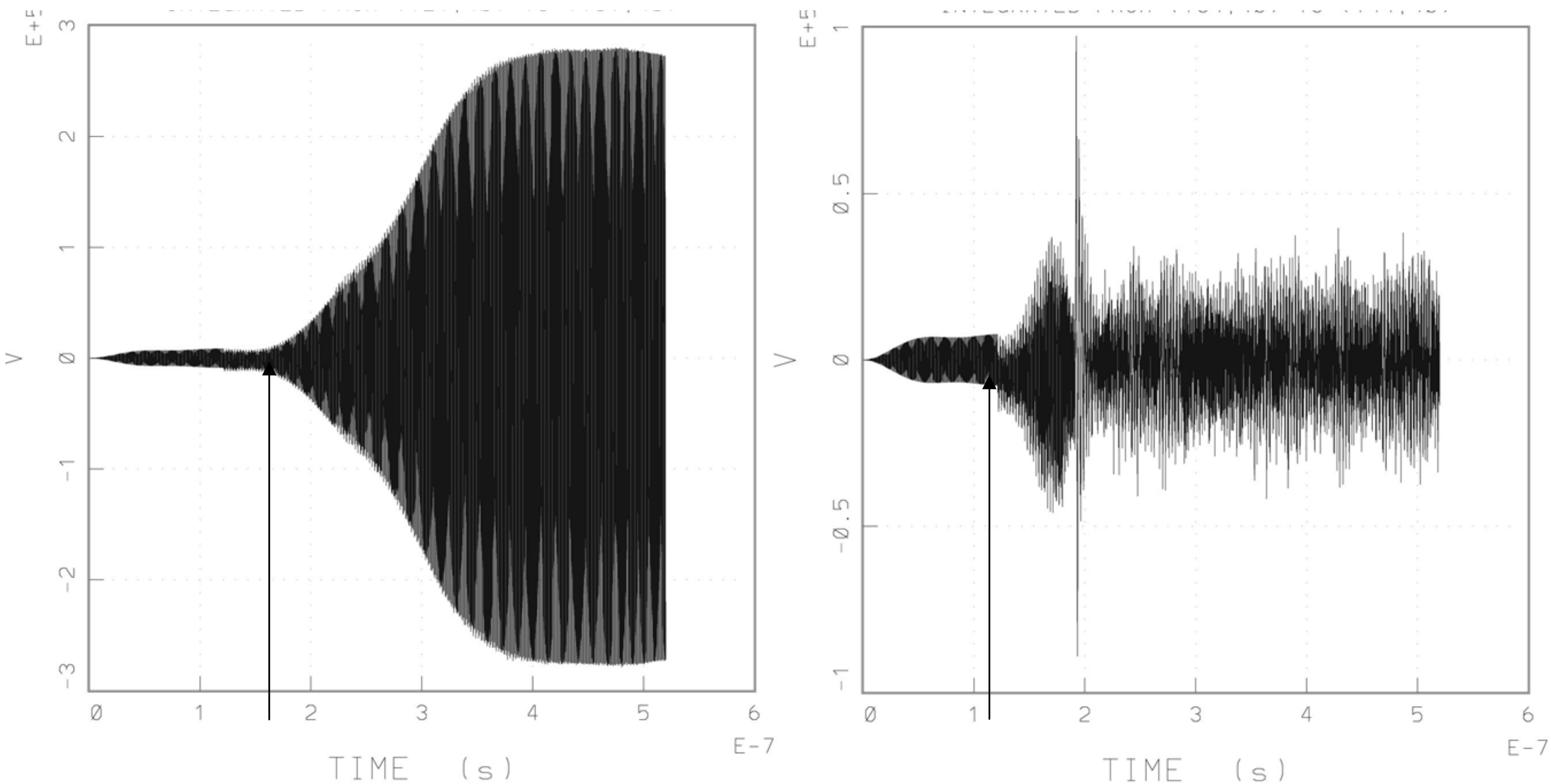
- **Selection:**
  - Exponential ranking
- **Mutation:**
  - Uniform mutation
  - Boundary mutation
  - Non-uniform mutation
  - Whole non-uniform mutation
- **Recombination:**
  - Whole arithmetic crossover
  - Simple arithmetic crossover

Results: Growth Rate "Optimized" Easily

- 50 independent experiments
  - 70 individuals
  - 500 generations
  - 2 evaluations per individual per generation
  - $\Rightarrow$ 3.5 million total evaluations
  - Wall clock time (233 MHz P-II, MS Windows NT 4.0) $\approx$ 1 hr
- Many experiments found high growth-rate designs near V_0 = 400 kV, I_0 = 25 ka, d = 9.4 cm
- Comparison to other designs:

V_0 (kV)	I_0 (kA)	d (cm)	Growth Rate
400	12	8.4	-1.87e6
600	24	8.4	-1.45e6
400	12	11.0	-2.76e6
400	25	9.4	-9.79e6

Results:
MAGIC Simulations



Simulation of EA's best design confirms high growth rate, predicts virtual cathode formation and corresponding shutoff

Methodology: Dispersion Relation Model of the Multi-cavity RKO

- Based on Luginsland's dispersion relation model of the two-cavity RKO (Luginsland, 1996)
- Models evolution of gap voltages including effects of:
  - Cavity resonances
  - Electromagnetic coupling
  - Beam coupling
- Assumptions
  - Small signal, modal, steady-state solutions
    - $\Rightarrow$ Superposition principle applies to beam modulation
  - Cavity coupling is weak and occurs through cutoff waveguide
    - $\Rightarrow$ Only nearest neighbor electromagnetic coupling is significant
- Generalizes model to the n-cavity RKO
  - Cavities may have distinct natural frequencies, qualities, and impedances
  - Drift regions may have distinct radii, lengths, and loss coefficients

Methodology:
Non-linear Multi-cavity RKO Model

Assuming solutions of the form $e^{-j\omega t}$, the gap voltage V_m satisfies

$$L_m(\omega)V_m + C_{m-1}V_{m-1} + C_m V_{m+1} + \sum_{n<m} \Gamma_{m,n} V_n = 0 \ ,$$

where the damped harmonic oscillator operator is

$$L_m(\omega) = \frac{\omega^2}{\omega_{0,m}^2} - \frac{j\omega}{\omega_{0,m} Q_m} - 1 \ ,$$

the electromagnetic coupling coefficient is

$$C_m = \chi_{c,m} \exp\left[-\frac{2.405}{r_{w,m}} \sqrt{1 - \left(\frac{2\pi\omega_{0,m} r_{w,m}}{0.383c} \right)^2} \left(x_{m+1} - x_m \right) \right] \ ,$$

and the beam coupling coefficient is

$$\Gamma_{m,n} = \frac{Z_m}{R} \sin\left\{ \sum_{r=n}^{m-1} k_{p,r} \left(x_{r+1} - x_r \right) \right\} \exp\left[-\frac{j\omega_{0,n}}{\beta c} \left(x_m - x_n \right) \right]$$

Methodology: Non-linear Multi-cavity RKO Model

- The evolution of the cavity voltages $V=(V_1, V_2, \ldots, V_N)^T$ is thus described by $[A(\omega)]V = 0$, where

$$A(\omega)=\begin{bmatrix} L_1(\omega) & -C_1 & 0 & \cdots & \cdots & \cdots & 0 \\ -\Gamma_{2,1}-C_1 & L_2(\omega) & -C_2 & 0 & \cdots & \cdots & 0 \\ -\Gamma_{3,1} & -\Gamma_{3,2}-C_2 & L_3(\omega) & -C_3 & 0 & \cdots & 0 \\ -\Gamma_{4,1} & -\Gamma_{4,2} & -\Gamma_{4,3}-C_3 & L_4(\omega) & -C_4 & & 0 \\ -\Gamma_{5,1} & -\Gamma_{5,2} & -\Gamma_{5,3} & -\Gamma_{5,4}-C_4 & L_5(\omega) & & 0 \\ \vdots & \vdots & \vdots & & & \ddots & -C_{N-1} \\ -\Gamma_{N,1} & -\Gamma_{N,2} & -\Gamma_{N,3} & -\Gamma_{N,4} & \cdots & -\Gamma_{N,N-1}-C_{N-1} & L_N(\omega) \end{bmatrix}$$

- Resonant frequencies ω satisfy $\det[A(\omega)] = 0$
  - $\det[A(\omega)]$ is a polynomial of degree $2N$ in ω
  - $-\operatorname{Im}[\omega]$ is the mode's growth rate, to be maximized

Methodology: Evaluation of Candidate Designs

- **Repair or penalize design if**
  - **Limiting current is exceeded**
  - **Beam radius (almost) exceeds waveguide radius**
- **Compute electromagnetic coupling coefficients (C's)**
- **Compute beam coupling coefficients (Γ's)**
- **Compute harmonic operator coefficients (L's)**
- **Construct the NxN matrix A(ω)**
  - **Elements are polynomials in ω, represented by their coefficients**

Methodology: Evaluation of Candidate Designs (Cont.)

- **Reduce $A(\omega)$ to lower triangular form:**
  - **For rows $i = N-1$ down to 1, and each element $[a(\omega)]_{i,j}$ in row i**
    - **Multiply by $[a(\omega)]_{i+1,i+1}$**
    - **Subtract $[a(\omega)]_{i,i+1}$ $[a(\omega)]_{i+1,j}$**
  - **det ($[A(\omega)]$) is now stored in $[A(\omega)]_{1,1}$ as a polynomial in ω of degree 2N**
- **Use Laguerre's method to find roots of det(a[ω)])**
- **Choose root ω s.t. $Re[\omega] > 0$ and $Im[\omega]$ is minimized**
- **Assign $Im[\omega]$ as the fitness of the candidate design**

Methodology: Non-linear Constraints

Check that drift space radius bounds satisfy constraints:

$$\left(0.95\frac{0.383c}{f_{0,m}}\right)-(r_o+0.2cm) \;\; \geq \;\; 0$$

Assuming constraint is satisfied, compute drift space radii:

$$r_{w,m}=\chi_{r,m}\left(0.95\frac{0.383c}{f_{0,m}}\right)+(1-\chi_{r,m})(r_o+0.2cm)$$

Check that limiting currents are not exceeded:

$$17000\left[\left(1+\frac{V_0}{mc^2}\right)^{\frac{2}{3}}-1\right]^{\frac{3}{2}}\left[1-2\left(\frac{r_i^2}{r_o^2-r_i^2}\log\frac{r_o}{r_i}-\log\frac{r_w}{r_0}\right)\right]^{-1}-I_0 \;\; \geq \;\; 0$$

Methodology: Handling Constraints

Standard EA techniques include

- Penalty functions
  - Assign zero fitness if limiting current is exceeded
  - Assign zero fitness if beam diameter greater than specified fraction of minimum waveguide diameter
- Repair operators (to evaluate or to replace)
  - Reduce current to limiting current
  - Reduce beam diameter to fit in waveguide
- Specialized mutation and recombination operators that maintain satisfaction of specified constraints
  - Computationally expensive in this application

Methodology: Handling Constraints in GENOCOP

If constraints are specified to match physical constraints:

- **All individuals in reference population satisfy specified constraints**
  - **Initial population therefore satisfies physical constraints**
- **Penalty function does not affect individuals that satisfy physical constraints**
  - **In particular, it does not affect reference population**
- **Operator that recombines search individuals with reference individuals must ensure specified constraints are satisfied**
  - **GENOCOP provides "blind" operators based on convex combinations**
  - **A domain specific operator can be used that changes only the necessary parameters of the search individual.**

Methodology: Handling Constraints in GENOCOP

If constraints are not specified:

- **Individuals in reference population are not constrained**
  - **Initial population is unlikely to contain any individuals that satisfy physical constraints**
- **Penalty function affects individuals that do not satisfy physical constraints**
  - **Initial population is likely to have all zero fitnesses**
- **Evaluation of individuals requires either a penalty function or a repair operator**
- **No real distinction between search and reference populations**

Methodology:
Handling Constraints

- **Eight methods of constraint handling used:**
- **Constraints**
  - **Unspecified, or**
  - **Specified to match physical constraints**
- **Evaluation of individuals not satisfying physical constraints**
  - **Penalized (zero fitness), or**
  - **Repaired (beam current and radius adjusted)**
- **Recombination of search and reference individuals**
  - **Blind convex operator, or**
  - **Heuristic operator (beam current and radius adjusted)**

Methodology:
Independent Variables and Domains

- Identify candidate designs, represented as vectors of independent variables:

$$(V_0, I_0, r_i, r_o\text{-}r_i,$$
$$f_{0,1},\ldots,f_{0,N},Q_1,\ldots,Q_N,Q_1Z_1,\ldots,Q_NZ_N,$$
$$d_1,\ldots,d_{N-1}, \chi_{r,1},\ldots, \chi_{r,N-1}, \chi_{c,1},\ldots, \chi_{c,N-1})^T$$

Quantity	Lower bound	Variable	Upper bound
Beam voltage	300 kV	V_0	650 kV
Beam current	5 kA	I_0	35 kA
Beam inner radius	0.1 cm	r_i	12 cm
Beam thickness	0.1 cm	$r_o - r_i$	1 cm
Cavity natural frequencies	1 GHz	f_0	2 GHz
Cavity qualities	50	Q	500
Cavity impedances	50 Ohms	QZ	377 Ohms
Drift space lengths	2 cm	d	50 cm
Drift space radius multipliers	0	χ_r	1
Drift space EM coupling multipliers	0	χ_c	1

Methodology: Computational Experiments

- **8 sets of experiments varying constraint handling**
- **For each constraint handling technique:**
  - **50 independent runs of 100,000 generations each**
  - **5 individuals in reference population**
  - **20 individuals in search population**
  - **2 evaluations per individual per generation $\Rightarrow$ 250 million total evaluations**
  - **Wall clock time (750 MHz P-III, Red Hat Linux 7) $\approx$ 14 hrs**

Results: High Growth-Rate, Non-intuitive, and Dissimilar Designs

- Each experiment found high growth-rate designs
  - For comparison, a 10 cavity version of one good two-cavity design has a growth rate of 1.30 $nsec^{-1}$
  - Best growth rate is 2.40 $nsec^{-1}$
  - Enhanced growth rates of 10-cavity design allow pure oscillator operation (two-cavity design requires injection-locked operation)
- Designs are non-intuitive (typical of EA-based design)
- Best designs from various experiments are dissimilar
  - Parameters differ significantly between cavities, and between drift spaces
  - Dissimilar results suggest the EA designs may be far from the global optimum (some similarity in beam voltage and cavity frequencies)

Results: Constraint Handling Techniques

- **Growth rates of best designs from each technique compared to other best designs using Kruskal-Wallis test**
  - **Heuristic recombination better than blind recombination (at 0.05 level of significance)**
  - **Repair to evaluate better than penalty function (at 0.10 level of significance**
- **Effective combinations:**
  - **Heuristic recombination and repair to evaluate**
  - **Specified constraints and heuristic recombination**
  - **Specified constraints, heuristic recombination, and repair to evaluate**
  - **Unspecified constraints, blind recombination, and repair to evaluate**
- **Ineffective combinations**
  - **Blind recombination and penalty function**
  - **Specified constraints and blind recombination**
  - **Specified constraints, blind recombination, and repair to evaluate**
  - **Specified constraints, blind recombination, and penalty**

RKO Circuit Model
Summary and Conclusions

- **Summary:**
  - **RKO circuit model and RKO growth rate model related through resonant frequencies to predict growth rate as a function of a limited set of design parameters : V_0, I_0, d**
  - **GENOCOP, a real-valued EA, using independent linear constraints on design parameters and standard algorithm parameters, identifies designs with growth rates that are significantly higher than previously investigated designs**
- **Conclusions:**
  - **Optimization of HPM device designs via EA is feasible**
  - **MAGIC simulations of EA-identified designs indicate that they are adversely affected by non-linear phenomena, such as virtual cathode formation**
  - **One method for treating non-linear phenomena is the specification of non-linear constraints**

RKO Dispersion Relation Model Summary

- **Theoretical model of signal growth rate in a multi-cavity RKO developed, incorporating electromagnetic and beam coupling effects**
- **Computational model manipulates arrays of polynomials to find determinant of interaction matrix, then uses Laguerre's method to find resonant frequencies and accompanying growth rates**

RKO Dispersion Relation Model Conclusions

- **GENOCOP, a real-valued EA, using independent linear constraints on design parameters and standard algorithm parameters, identifies designs with growth rates that are significantly higher than intuitive designs**
- **A version of GENOCOP with a domain specific repair operator for recombination of search and reference individuals identifies even better designs**
- **Design optimization via EA pays off in two ways**
  - **Better designs**
  - **Improved understanding of models**

Future Directions

- Improve diagnostic output of GENOCOP
  - Fitness statistics (for search and reference)
  - Diversity measures (for search and reference)
- Improve effectiveness and efficiency of optimization
  - Hybridize with local search (e.g. conjugate gradient)
  - Consider other optimum-seeking techniques
  - Reduce the number of roots found
- Improve theoretical and computational models
  - Consider limiting currents at cavity gaps
  - Consider mode competition and sensitivity to design parameters
- Perform PIC simulations of best designs

References

- Hendricks, Coleman, Lemke, Arman, and Bowers, *Physical Review Letters*, vol 76, no 154, 1996.
- Luginsland, Lau, Hendricks, and Coleman, *IEEE Transactions on Plasma Science*, vol 24, no 3, 1996.
- Michalewicz, Genetic Algorithms + Data Structures = Evolution Programs, Springer-Verlag, New York, 1992.
- Schiffler, Luginsland, Hendricks, and Spencer, AFRL Technical Report, 1998.